Journal of Applied Mechanics and Technical Physics, Vol. 35, No. 3, 1994

MODELING THE AERODYNAMIC DRAWING OF A VISCO-ELASTIC FLUID BY A THIN NONISOTHERMAL JET

A. L. Kalabin

UDC 532.526

Research into the subject of the aerodynamic drawing of melts has found application in the development of a technology for the aerodynamic forming of chemical fibers (AFF). In AFF, the drawing force is the aerodynamic frictional force between the fiber and air $F_{ar}(x)$ generated by an ejector that creates a flow of air along the forming fiber. When used in the production of nonfabric materials from a melt, the method makes it possible to obtain fibers and the final product in a single step. The problem of fiber formation was first formulated in [1], while the theory of fiber formation was analyzed in [2, 3]. The authors of [4-6] examined some of the technical and physical aspects of AFF. The well-known mathematical models of AFF [7, 8] cannot be used for several reasons. First of all, the theoretical relation employed to characterize the aerodynamic force $F_{ar}(x)$ along the path on which the fibers are formed in AFF [7] is not consistent with qualitative analyses made on the basis of empirical data on fiber and air velocities [4-6]. Secondly, use of the ratio from [7] to calculate $F_{ar}(x)$ leads to first-order discontinuities of the function $F_{ar}(x)$ at points on the formation path where air and fiber velocity are equal. The discontinuities result in a sudden change in the sign and magnitude of the aerodynamic force at these points, which is inconsistent with representations of the continuity of the acting force. No relation was presented in [8] to calculate the aerodynamic force when an ejector is used in the drawing process. Also, no previous research has considered the choice of boundary conditions for the equation of motion. In AFF, the pulling force that is the main force determining the motion of the fluid is the force of interaction of the stream and the accompanying flow of air $F_{ar}(x)$. Its magnitude depends on the velocity of the moving stream v(x), which is an unknown in the equation of motion. We thus encounter a problem in which the boundary condition depends on the solution of this equation. This makes it necessary to have an algorithm for selecting the correct boundary conditions to describe the motion of a stream of viscoelastic fluid. Below, we propose a mathematical model to describe the aerodynamic drawing of a thin stream of viscoelastic fluid.

Basic Equations and Boundary Conditions. The balance of the forces acting on the stream as it moves has the form [1, 2]

$$F_{rh}(x) = F_{rh}(0) + F_{in}(x) + F_{ar}(x) + F_{st}(x) - F_{gr}(x), \tag{1}$$

where

$$F_{rh}(x) = p_{xx}S \tag{2}$$

is the rheological force; $p_{xx} = \mu(T)\, dv/dx$ is the tensile stress; S is the cross-sectional area of the stream; v is the velocity of the stream; $\mu(T, dv/dx)$ is the viscosity of the polymer;

$$F_{gr}(x) = \frac{\pi}{4} g\rho \int_0^x D^2 dx \tag{3}$$

is the gravitational force; g is gravitational acceleration; ρ is the density of the polymer; D is the diameter of the stream;

$$F_{ar}(x) = \pi \int_0^x \mathrm{sign}(\Delta v) p_{xx} D dx \tag{4}$$

Tver'. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 35, No. 3, pp. 112-116, May-June, 1994. Original article submitted June 15, 1993.

0021-8944/94/3503-0425$12.50

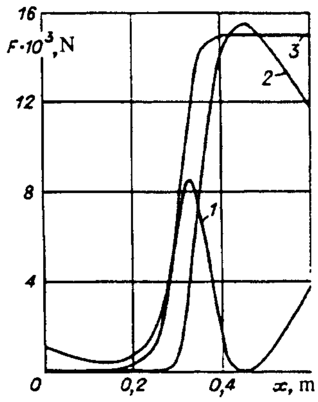

Fig. 1

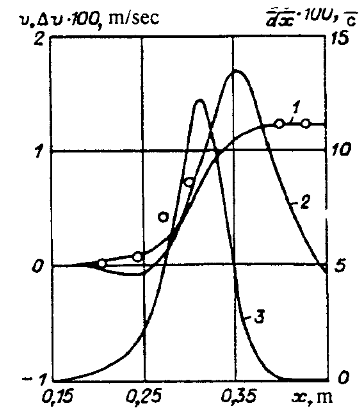

Fig. 2

is the aerodynamic force; $p_{xs} = 0.5c_f\rho_0\Delta v^2$ is the shear stress; $c_f = a_c Re^{-\xi}$ is the coefficient of aerodynamic friction; ρ_0 is the density of the air; $\Delta v = v - u$ is the difference between the velocities of the stream v and air u; Re is the Reynolds number; a_c and ξ are constants;

$$F_{sf} = \frac{\pi}{2}\sigma(D_0 - D); \tag{5}$$

σ is the coefficient of surface tension; D_0 is the diameter of the stream at x = 0;

$$F_{in} = G(v - v_0) \tag{6}$$

is the inertial force; v_0 is the initial velocity of the stream;

$$G = \rho v S \tag{7}$$

is the continuity equation; G is the rate of flow of the polymer.

The equation for the aerodynamic force (4) differs from that used in [1-3] in the presence of the function sign (Δv), since this force is alternating in the given case. In contrast to [7], we put the function sign(Δv) in the integrand in (4) to obtain qualitative agreement between the calculated value of $F_{ar}(x)$ and the empirical data on stream and air velocities and ensure that the directions of change in these quantities in (4) are correct.

In accordance with [9], we take the dependence of the viscosity of the polymer on temperature in the form

$$\mu = \mu_0\theta^{-\beta}, \tag{8}$$

where μ_0 is the viscosity at the initial temperature T_0; $\beta >> 1$; $\theta = (T - T_*)/(T_0 - T_*)$; T_* is the glass point.

Having differentiated Eq. (1) and having inserted (2-8) into the result, we obtain the equation of motion of the stream:

$$\frac{d^2v}{dx^2} - \left(\frac{1}{v}\frac{dv}{dx} + \frac{\beta\theta^{-1}}{T_0 - T_*}\frac{dT}{dx} + v\theta^\beta\,[A_{in} + A_{sf}v^{-1.5}]\right)\frac{dv}{dx} - \theta^\beta[A_{ar}\mathrm{sign}(v)\Delta v^{0.5(\xi-1)}v^{3-\xi} - A_{gr}] = 0. \tag{9}$$

Here

$$A_{ar} = \frac{a_c\rho_0}{\mu_0}\left(\frac{2}{v_0}\right)^{-\xi}\left(\frac{\pi\rho}{G}\right)^{0.5(1+\xi)};$$

$$A_{in} = \rho/\mu_0;\quad A_{gr} = A_{in}g;\quad A_{sf} = \frac{\sigma}{2\mu_0}\sqrt{\frac{\pi\rho}{G}}.$$

In accordance with [1-3], the equation describing heat transfer between the stream and air has the form

$$\frac{dT}{dx} = -\frac{\pi\alpha D}{GC}(T - T_s), \tag{10}$$

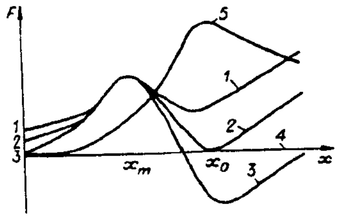

Fig. 3

where $\alpha = \lambda_0 a_n \mathrm{Re}^{\gamma}/D$ is the heat-transfer coefficient calculated from the relation $\mathrm{Nu} = a_n \mathrm{Re}^{\gamma}$ [2]; λ_0 is the thermal conductivity of air; C is the heat capacity of the polymer; T_s is air temperature.

System (9-10) is a mathematical model of the motion of the stream. Nonlinear system of ordinary differential equations (9-10) was solved by a fourth-order Runge—Kutta method with a constant step.

The following values of process parameters were taken from [4-6] for numerical modeling: radius of the spinneret $R^0 = 0.25$ mm; flow rate $G = 1$ g/min; distance from the ejector to the spinneret $x^0 = 30$ cm. The properties of the polypropylene melt were taken mostly from [9]: $\mu_0 = 291$ kg/(m·sec), $T_0 = 573$ K, $T_* = 323$ K, $\rho = 800$ kg/m^3, $C = 2400$ J/(kg·K), $\beta = 8.4$. We used data from [10] for the air: density $\rho_0 = 1.29$ kg/m^3, kinematic viscosity $\nu_0 = 1.4 \cdot 10^{-7}$ m^2/sec, thermal conductivity $\lambda_0 = 0.04$ W/(m·K). For Eq. (10), we took the constants $a_c = 0.9$ and $\xi = 0.4$ from [2] and $a_n = 0.4$ and $\gamma = 0.3$ from [6]. We chose the empirical functions from [4, 5] for the temperature $T_s(x)$ and velocity u(x) of the air.

Analysis of the Dynamics of Motion. For the numerical modeling, we first determined the initial velocity from continuity equation (7) by means of the formula

$$v(0) = 4G/(\pi D_0^2 \rho), \tag{11}$$

while the second boundary condition (gradient of velocity v′(0) at x = 0) was varied so that the "final" velocity v(L) changed from 25 to 200 m/sec.

To analyze the dynamics of motion of the stream, we will present the results of calculations of the forces acting on the stream. These calculations were performed using Eqs. (2-6). Figure 1 shows the typical distribution of these forces along the formation path x(1 — F_{rh}, 2 — ($-F_{ar}$), 3 — F_{in}). The calculations indicated that surface tension and the gravitational force are no greater than 6 and 12% of the maximum rheological force, respectively, which shows that they are small. Of the greatest interest is the behavior of the aerodynamic force, since it is the main factor determining the motion of the stream. The dependence of this force on the coordinate x has two local extrema, their positions corresponding to the coordinates where the difference in the velocities of the stream and air Δv changes sign. This occurs at x = 0.26 and 0.45 m, according to Fig. 2. In the Fig. 2, 1 represents v, 2 represents ($-\Delta v$), and 3 represents dv/dx. The change in the sign of the integrand in (4) corresponds to an extremum of the primitive of this function — which in the given case is the aerodynamic force. The change in the inertial force along the coordinate x is monotonic and agrees qualitatively with the graph of fiber velocity (Fig. 2). The behavior of the forces as coordinate functions agrees qualitatively with the experimental data presented in [1, 2].

It follows from the results of the modeling that with a "final" velocity v(L) > 50 m/sec, we can analyze the motion of the stream while ignoring the contribution of the gravitational force and surface tension. As a result, Eq. (1) takes the form

$$F_{rh}(x) = F_{rh}(0) + F_{in}(x) + F_{ar}(x); \tag{12}$$

After differentiating (12), we obtain the approximate equation of motion of the stream:

$$F'_{rh}(x) = F'_{in}(x) + F'_{ar}(x) \tag{13}$$

(the prime denotes differentiation with respect to x). This relation represents the balance of the forces acting on an infinitesimal element of stream length dx.

Selection of Boundary Conditions. The numerical modeling allowed us to determine the qualitative dependence of the rheological force $F_{rh}(x)$ on the coordinate x, which is shown in Fig. 3 [curves 1-4 show the rheological force $F_{rh}(x)$ at $v_1'(0) > v_2'(0) > v_3'(0)$, where subscripts 1, 2, and 3 correspond to the numbers of the curves and curve 5 shows the aerodynamic force $F_{ar}(x)$].

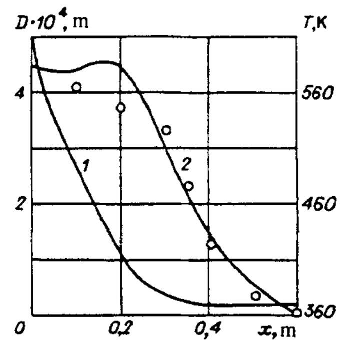

Fig. 4

The numerical modeling showed that $F_{rh}(x)$ always has a local maximum $F_{rh}(x_m)$ and a local minimum $F_{rh}(x_0)$. At the beginning of the stream, $F_{rh}(x)$ is determined by the initial value $F_{rh}(0)$. There is then a local maximum $F_{rh}(x_m)$, which is necessarily followed by the local minimum $F_{rh}(x0)$. The point of the minimum $F_{rh}(x_0)$ $x = x_0$ coincides with the point of the maximum $F_{ar}(x_0)$, where the conditions for the extrema of the aerodynamic and rheological forces $F_{ar}'(x_0) = 0$ and $F_{rh}'(x_0) = 0$ are satisfied. As a result of these conditions and Eq. (13), we find that $F_{in}'(x_0) = 0$. However, it is known from the relations for calculating forces which was presented in [2] that $F_{in}'(x) = \rho v\, F_{rh}(x)/\mu$. It in turn follows from the last equation that the rheological force at point x_0 is equal to zero: $F_{rh}(x^0) = 0$. Inserting the last relation into (12), we obtain the following force balance at point $x = x_0$

$$F_{rh}(0) = -[F_{in}(x_0) + F_{ar}(x_0)],$$

With allowance for the formula used to calculate the rheological force $F_{rh}(x) = \pi R^2 \mu v'(x)$, we can then use the expression just presented to determine the velocity gradient at the initial coordinate $x = 0$:

$$v'(0) = -[F_{in}(x_0) + F_{ar}(x_0)]/(\pi R_0^2 \mu). \tag{14}$$

The value of x_0 is found from the equation $\Delta v = v(x) - u(x) = 0$, since it follows from (4) that F_{ar}' is proportional to Δv and that $F_{ar}'(x^0) = 0$ at this point.

Equation (14) makes it possible to take a set of functions $F_{rh}(x)$ (curves 1-3 in Fig. 3) obtained with different initial gradients $v'(0)$ and choose the rheological force (curve 2 in Fig. 3) that corresponds to the motion of the stream under the aerodynamic force alone. The other functions $F_{rh}(x)$ (curves 1 and 3 in Fig. 3) correspond to cases in which the stream is also acted upon by an external tensile force.

Based on the above analysis, it is recommended that the path of the stream [0, L] be broken up into two intervals along the coordinate. In calculating motion in the first interval $[0, x_0]$, the second boundary condition is chosen numerically (by an iterative procedure) in accordance with Eqs. (11) and (14). The boundary conditions in the second interval $[x_0, L]$ are the final velocity in the first interval $v(x_0)$ and the velocity gradient, which is equal to zero at x_0. Figure 3 (curve 4) shows the rheological force calculated by this approach.

Numerical Modeling. The distributions of velocity, the longitudinal velocity gradient, and the diameter and temperature of the stream play the main roles in determining the structure of the polymer fibers [2, 3], so we used the proposed model to calculate these quantities. They were obtained by solving system (9-10). The results of the calculations are shown in Figs. 2 and 4. Lines 1-3 in Fig. 2 show the dependence of fiber velocity v, the difference between the velocities of the air and fibers Δv, and gradient of fiber velocity dv/dx on the coordinate x. Figure 4 shows the distribution of stream diameter D and temperature T (lines 1 and 2) along x. For comparison, Figs. 2 and 4 show experimental data from [4-6]. Results calculated with the proposed mathematical model and the experimental data agree satisfactorily, which demonstrates the adequacy of the model.

The chosen boundary conditions make it possible to calculate the parameters of the stream for different conditions of its motion.

REFERENCES

1. S. Kase and T. Matsuo, "Studies on melt spinning. 1. Fundamental equations on the dynamics of melt spinning," J. Polym. Sci., **A3**, No. 7 (1965).
2. A. Zyabitskii, Theoretical Principles of Fiber Formation [in Russian], Khimiya, Moscow (1979).
3. Dei Han Chang, Rheology in Polymer Processing Operations [Russian translation], Khimiya, Moscow (1979).
4. A. V. Genis, D. V. Fil'bert, and A. A. Sindeev, "Aerodynamic formation of fibers from a melt," Khim. Volokna, No. 1, 7-10 (1978).
5. A. V. Genis, D. V. Fil'bert, and A. A. Sindeev, ibid., No. 3, 27-29 (1978).
6. A. V. Genis, D. V. Fil'bert, A. A. Sindeev, and O. G. Korovitsyn, "Cooling of a fiber during the aerodynamic forming of fibers from polymer melts," ibid., No. 5, 30-32 (1978).
7. N. K. Zhiganov, V. I. Yankov, E. I. Alekseev, and A. V. Genis, "Mathematical model of the aerodynamic method of forming fibers from polymer melts," ibid., No. 4, 18-20 (1988).
8. V. I. Eliseev and L. A. Fleer, "Coupled problem concerning the aerodynamic drawing of a stream of heated viscous liquid," Prikl. Mekh. Tekh. Fiz., No. 6, 148-153 (1988).
9. D. V. Ivanyukov and M. L. Fridman, Polypropylene, Khimiya, Moscow (1974).
10. Tables of Physical Quantities. Handbook [in Russian], Atomizdat, Moscow (1976).